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Short communication

Reinvestigation on Assessing the Stability of Mullagulov Tested Steel Rods under Follower Forces

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ABSTRACT

Dynamic instability is an interesting topic in the mechanics of elastic structures. Though the subject has been formed by many analytical, numerical, and experimental investigations, it has many issues, as evidenced from the critical overview of Elishakoff. Furthermore, the controversial articles of Koiter and Sugiyama on unrealistic and realistic follower forces demand experimental verification. Mullagulov has proposed a device for creating the follower forces and tested steel rods under compression. This paper highlights the experimentation of Mullagulov and his observations briefly to examine the influence of material properties on the stability load estimations and to confirm the practical realization of follower forces.

Keywords: Beck column, coalescence frequency parameter, concentrated tangential load, critical load parameter

INTRODUCTION

Elastic structures under non-conservative forces are subject to flutter or dynamical instability. Dissipation plays a fundamental and destabilizing role in these structures. A large difference is noticed in critical load assuming the absence of dissipation and limit of vanishing dissipation ('the Ziegler paradox'). This subject finds importance in structural mechanics, physics, rotor dynamics, aeroelasticity, and fluid-structure interaction (Mirko, 2018). Space launch vehicles under aerodynamic (drag) forces are modeled as columns under compressive loads, whose structural integrity is assessed by performing an overall stability analysis for the flight conditions.

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The static and dynamic stability of systems loaded with flower forces has attracted many researchers to investigate and contribute. Von Beck has solved analytically the standard problem (namely, a cantilever column subjected to follower force) for the first time. Hence, the problem is referred to as the famous Beck column. Analytical, numerical, and experimental studies are made (after Beck in 1952) on a cantilever column with a tip-concentrated tangential or sub-tangential follower load (Anderson & Thomsen, 2002; Langthjem & Sugiyama, 2000a; Langthjem & Sugiyama, 2000b; Rao & Rao, 1989a; Rao & Rao, 1989b; Rao & Rao, 1990; Rao & Rao, 1991; Madhusudan et al., 2003; Zahharov et al., 2004; Kwasniewsi, 2010; Mutyalarao et al., 2012). The controversial articles of Koiter (1996) and Sugiyama et al. (1999) on unrealistic and realistic follower forces remain a matter of debate (Mascolo, 2019).

Timoshenko and Gere (2012) have emphasized experimental validation. Sugiyama et al. (2000), Sugiyama (2002), and Sugiyama et al. (2019) have performed experiments on cantilever columns under a tip-concentrated sub-tangential follower force by mounting a solid rocket motor at free-end. They have demonstrated the stabilization of the system due to rocket thrust. However, their test results cannot be utilized directly to compare critical load estimates (Mutyalarao et al., 2017). Tomski and Uzny (2008) have examined a slender system with force directed toward the positive pole, which falls under the conservative load category. They have also considered a non-conservative load case, in which Beck's load is generated through a reaction engine.

Willems (1966) has adopted a simple procedure and performed experiments. Though the configuration has a critical load close to Beck's problem, theoretical analysis of Huang et al. (1967) and discussion of Augusti et al. (1967) have concluded that Willems test results are not representing the Beck column. Praveen et al. (2021) have revisited the critical load assessment of Huang et al. (1967) and corrected the error in the computed values and confirmed the adequacy of Willems's (1966) approach on the Beck column. Xiong et al. (1989) have considered a centripetally loaded model and simulated Beck's column showing equivalence of the first and second modes individually, whereas Praveen et al. (2020) have demonstrated the equivalence of the first and second modes directly from the dynamic characteristics of the Beck column. The discrepancy in the test data and the dynamic analysis results can be due to assigned values of Young's modulus and density of the material. Praveen et al. (2020) commented on the analysis of Xiong et al. (1989) that there is no necessity to modify the characteristic equation of the Beck column to match the test data. Xiong et al. (1989) have not conducted experiments up to the initiation of instability.

Mullagulov (1994) has made an interesting experimental-theoretical study on the stability of rods compressed by follower forces. This paper briefly presents the critical load evaluation of Beck column, Mullagulov experimentation, and his observations to examine the influence of material properties on the stability load estimates and confirm follower forces' practical realization.

METHODS

Critical Load Evaluation of Beck Column

Based on the moment-curvature relationship, Mutyalarao et al. (2017) have employed a system of non-linear ordinary differential equations assuming harmonic motion to investigate cantilever columns' stability under a sub-tangential follower force. Figure 1 depicts the tip-angle $\phi(0)$ and a follower force P with load rotation (or sub-tangential) parameter β. OB and OCA are the un-deformed and deformed positions. The column length is L. (X_a, Y_a), are the tip-coordinates and $(u,v) = (X - L + s,\ Y)$, are the deflections. The angle $\phi(s)$ is between the tangent to the bent axis and the vertical. Deflection curve length (s) of the column is from its tip. The load rotation parameter, $\beta = 0$ represents the Euler's column, in which P acts in vertical direction, whereas, $\beta = 1$ represents the Beck's column, in which P acts as a tangential force.

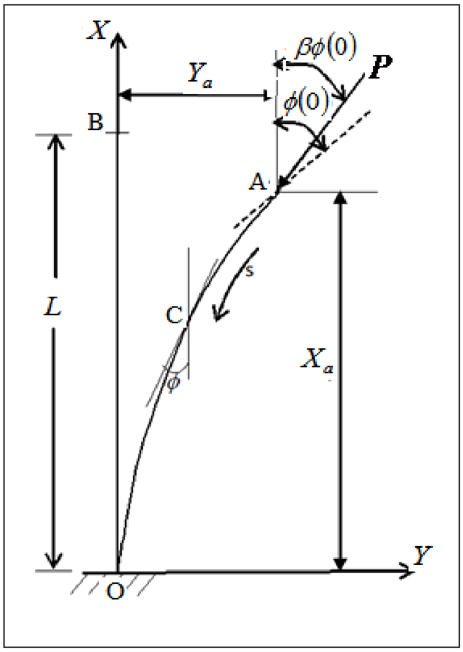

Figure 1. Cantilever column under a tip-concentrated sub-tangential follower force

Mutyalarao et al. (2017) have defined $x = \dfrac{X}{L}$; $y = \dfrac{Y}{L}$; $\eta = \dfrac{s}{L}$; load parameter, $\lambda = \dfrac{PL^2}{EI}$; frequency parameter, $\omega = \Omega L^2 \sqrt{\dfrac{m}{E}}$ and presented the nonlinear differential Equations 1-5 in non-dimensional form as

$$\phi'' + \lambda \sin(\phi - \beta\phi(0)) + \omega^2 (H \sin\phi + V \cos\phi) = 0 \tag{1}$$

$$H' - (1 - \eta - x) = 0\,, \tag{2}$$

$$V' - y = 0\,, \tag{3}$$

$$x' + \cos\phi = 0\,, \tag{4}$$

$$y' + \sin\phi = 0\,, \tag{5}$$

Here prime denotes differentiation with respect to η . *m*, is the mass per unit length of the column. Ω, is the circular frequency. *E*, is the Young's modulus and *I*, is the moment of inertia. The second, third and fourth terms in Equation 1 are due the bending moments produced by the tip –concentrated sub tangential follower load the action of inertia forces. The boundary conditions for Equations 1-5 are as below (Equations 6-7)

$$\phi = \phi(0)\ ,\ \phi' = H = V = 0 \quad \text{at } \eta = 0 \tag{6}$$

$$\phi = x = y = 0 \ \text{ at } \eta = 1 \tag{7}$$

For the case of small deflections ($\phi \to 0$), replacing $\frac{\phi}{\phi(0)}$ *as* $\tilde{\phi}$ and $\frac{y}{\phi(0)}$ *as* $\tilde{y}$, the nonlinear ordinary differential Equations are reduced to (Equation 8)

$$\tilde{y}^{iv} + \lambda\,\tilde{y}'' - \omega^2\tilde{y} = 0 \tag{8}$$

The boundary conditions are as in Equations 9-10

$$\tilde{y}' = -1,\ \tilde{y}'' = 0,\ \tilde{y}''' = \lambda(1-\beta) \ \text{ at } \eta = 0 \tag{9}$$

$$\tilde{y} = \tilde{y}' = 0 \ \text{ at } \eta = 1 \tag{10}$$

The general solution of the Equation 8 is as stated in Equation 11

$$\tilde{y}(\eta) = A\cosh(\lambda_1\eta) + B\ \sinh(\lambda_1\eta) + C\ \cos(\lambda_2\eta) + D\sin(\lambda_2\eta) \tag{11}$$

Here $A = -\frac{(B\sinh\lambda_1 + D\sin\lambda_2)}{(\cosh\lambda_1 + \lambda_1^2\lambda_2^{-2}\cos\lambda_2)}$; $B = \frac{\lambda(1-\beta) - \lambda_2^2}{\lambda_1(\lambda_1^2 + \lambda_2^2)}$; $C = \left(\frac{\lambda_1}{\lambda_2}\right)^2 A$; $D = -\frac{\lambda(1-\beta) + \lambda_1^2}{\lambda_2(\lambda_1^2 + \lambda_2^2)}$; $\lambda_1 = \sqrt{-0.5\lambda + \sqrt{\omega^2 + 0.25\lambda^2}}$; and $\lambda_2 = \sqrt{0.5\lambda + \sqrt{\omega^2 + 0.25\lambda^2}}$.

The transcendental equation relating to load parameter (λ) and the frequency parameter (ω) is of the form Equation 12

$$\beta\lambda^2 + 2\omega^2 + \left(\lambda^2(1-\beta) + 2\omega^2\right)(\cosh\lambda_1\cos\lambda_2) + (2\beta - 1)\lambda\omega\sinh\lambda_1\sin\lambda_2 = 0 \tag{12}$$

Equation 12 is solved for ω specifying λ and β. Figure 2 shows the variation of ω with λ for different values of β ranging from 0 to 1 with an incremental step size of 0.1.

The Eigencurves of the column (namely, the load parameter (λ) versus frequency parameter (ω) curves) indicate the stability of the equilibrium position of the column. Static stability loads are those at which the Eigencurve meets the load (λ) axis. The dynamic stability loads are those at which two branches of Eigencurves coalesce. Static stability loads are possible when the sub-tangential parameter $\beta = \frac{1}{2}$. For $\beta = \frac{1}{2}$, the first load-frequency curve touches the load axis and later it coalesces with the second Eigencurve of the column. For $\frac{1}{2} < \beta < 1$, the two branches of Eigencurves coalesce. From the Eigencurves in Figure 2, it is noticed that the transition of static and dynamic stability takes place at $\beta = \frac{1}{2}$. For Euler's column ($\beta = 0$), the critical load parameter, $\lambda_c = 2.4674$, whereas, $\lambda_c = 20.05$ and the coalescing frequency parameter, $\omega_c = 11.01$ for Beck's column ($\beta = 1$). These results are in good agreement with the numerical solutions (Mutyalarao et al., 2012).

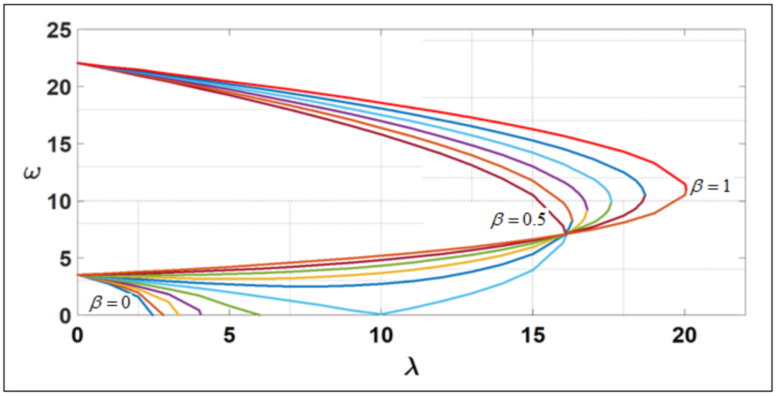

Figure 2. Variation of frequency parameter (ω) with load parameter (λ) for different sub-tangential parameters (β) ranging from 0 to 1 with an incremental step size of 0.1

RESULTS AND DISCUSSION

Mullagulov Experimentation on Creation of Follower Force

Figure 3 shows a schematic of Mullagulov (1994) stand to create the follower force for the rods through a propeller driven by a high- speed de electric motor (having 2.6 kg weight; 1.47 kW power; and the rotational speed limiting to 6000 rpm).

The test rod is mounted horizontally on the stand base and installed with an electric motor (with a propeller). The electric motor is suspended on a ceiling wire to suppress the bending of the test rod from the motor weight. The influence of torque is eliminated by mounting a special carriage at the end of the test rod. The hardened and ground surfaces of the guides are lubricated with machine oil for making the free end of the rod displace freely in the horizontal plane without friction. A balancing device is provided to eliminate the influence of restoring forces (if any) due to the attachment of the electric motor to the test rod, keeping the same length of ceiling wire and the rocker-strut.

Gradual increasing of the electrical voltage increases the compressive follower force smoothly. When it reaches the critical value, oscillations of the test rod with increasing

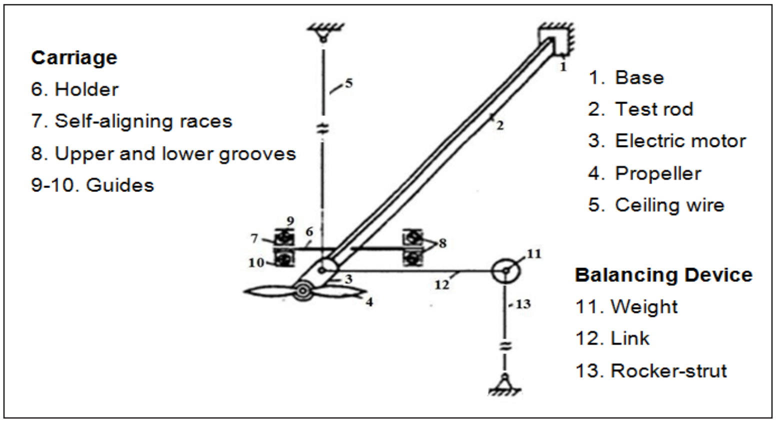

Figure 3. Schematic of stand for creating a follower force (Mullagulov, 1994)

amplitude begin spontaneously without any artificial disturbance. The stand is quite simple in design and fabrication and has no restriction on its operation time. The stability of the column is assessed from the load versus frequency curve (which is nothing but the Eigencurve). The dynamic stability load of a Beck column is the minimum load where two branches of Eigencurve coalesce. Analytical and numerical results on the Beck column are presented in dimensionless parameters. The load parameter (λ) and the frequency parameter (ω) for the Beck column defined are as in Equation 13-14.

$$\lambda = \frac{P L^2}{E I} \tag{13}$$

$$\omega = \Omega L^2 \sqrt{\frac{m}{E}} \tag{14}$$

Here, E is the tip-concentrated follower load; E is the Young's modulus; $I \left(= \frac{b d^3}{12} \right)$, is the moment of inertia; L is the column length; b and d are column breadth and depth respectively; m is the mass per unit length of the column; and Ω is the circular frequency. The critical load parameter (λ_c) and the coalescing frequency parameter (ω_c) for the Beck column obtained are 20.05 and 11.01 respectively. The critical load (P_{cr}) for the Beck column can be estimated from Equation 15.

$$P_{cr} = \frac{\lambda_c \times E \times I}{L^2} = 20.05 \times \frac{E \times I}{L^2} \tag{15}$$

It should be noted that the critical load parameter (λ_c) arrived from the dynamic stability of a Beck column (A uniform cantilever column under a tip-concentrated follower load) is 20.05. The critical load (P_{cr}) for the Beck column in Equation 15 is directly proportional to the stiffness of the column (E) and inversely proportional to the square of the column length (L^2). Russian Professors/ Scientists have been performed experiments on cantilever hardened steel rods repeatedly in several years. The rods having constant rectangular cross- section with dimensions: Length, L =1000 mm; Width, b =39.4 mm; and Depth, d = 0.97 mm. Young's modulus of the hardened steel varies from 190 to 210 GPa. Hence, the bending stiffness of the rod EI varies from 0.57 to 0.63 N-m^2. Table 1 gives the critical load of the hardened steel cantilever rod subjected to a tip- concentrated follower force. For the variation of bending stiffness, the estimated range of the critical load (P_{cr}) is from 11.43 to 12.63 N for the Beck column from Equation 15. The test results in Table 1 are within the expected range.

Table 1
Experimental investigations of Russian Professors/Scientists on a cantilever hardened steel rod subjected to a tip-concentrated follower force

Professor/Scientist	Critical load, $P_{cr}(N)$	
	Test (Mullagulov, 1994)	Estimated range (Eq.15)
K. G. Galimkhanov	12.0	11.43 - 12.63
G. S. Shpire	11.8	
I. Kh. Khairullin	11.7	
M. I. Vil'danov	11.6	
G. Batyrgareev	12.1	
M. Kh. Mullagulov	12.2	

Mullagulov (1994) performed experiments on hardened steel rods with different lengths and rectangular cross-sections to record the critical follower load. Table 2 presents the critical follower loads for the rod lengths varying from 0.7 to 1 m with a different moment of inertia (varying rectangular cross-section). For a constant stiffness of the column, the critical follower load (P_{cr}) decreases with increasing column length. In the case of a fixed column length, the critical follower load (P_{cr}) increases with increasing the stiffness of the column. The test results in Table 2 are found to be within the expected range.

Assuming all the hardened steel rods tested by Mullagulov (1994) are from the same batch, one test result is used to evaluate Young's modulus (E). From Equation 15, one can find Young's modulus (Equation 16),

$$E = \frac{P_{cr} \times L^2}{\lambda_c \times I} = \frac{11.8 \times 1^2}{20.05 \times 3 \times 10^{-12}} = 1.96176 \times 10^{11}\, N/m^2 \approx 196.2\, GPa \qquad (16)$$

The Young's modulus of the hardened steel rods is worked out to be 196.2 *GPa*, which is within the variation of 190 to 210 *GPa*. The critical load estimates in Table 2 using Young's modulus value of 196.2 *GPa* are close to the test results (Mullagulov, 1994).

Table 2
Comparison of experimental and analytical flutter loads of rods having a constant rectangular cross-section with different lengths and bending stiffness

Length, L (m)	Flutter or Critical Load, $P_{cr}(N)$		
	Test	Present Analysis	
	(Mullagulov, 1994)	E = 190 – 210 GPa	E = 196.2 GPa
Moment of inertia, $I = 3 \times 10^{-12}\ m^4$			
0.7	24.8	23.32 – 25.78	24.08
0.8	19.0	17.86 – 19.74	18.44
0.9	14.8	14.11 – 15.59	14.57
1.0	11.8	11.43 – 12.63	11.80
Moment of inertia, $I = 2.35 \times 10^{-12}\ m^4$			
0.7	19.2	18.27 – 20.19	18.87
0.8	14.6	13.99 – 15.46	14.44
0.9	11.5	11.05 – 12.22	11.41
1.0	9.2	8.95 – 9.89	9.24
Moment of inertia, $I = 1.62 \times 10^{-12}\ m^4$			
0.7	13.2	12.59 – 13.92	13.01
0.8	10.0	9.64 – 10.66	9.96
0.9	8.0	7.62 – 8.42	7.87
1.0	6.23	6.17 – 6.82	6.37

CONCLUSION

Many researchers have made interesting investigations on the static and dynamic stabilities of the systems loaded with follower forces. Since the critical load of the Beck column (namely, a cantilever column subjected to a tip—concentrated tangential load) is found to be nearly eight times that of the classical Euler column, there is a demand for experimental validation. Willems (1966), Xiong et al. (1989), and Sugiyama et al. (2000) have been partially successful in performing the experiments.

Mullagulov (1994) successfully created the cantilever rod's follower force and performed tests. This paper reinvestigates on the stability assessment of Mullagulaov tested steel rods under follower forces and the influence of material properties on the critical load evaluation of the Beck column. One of his test results is considered and evaluated Young's modulus of the hardened steel as 196.2 GPa, which is used for critical load evaluation of other cantilever rods having different lengths and bending stiffness. Estimates of the critical load match well with test results. This study indicates the practical realization of follower forces through a simple design and fabrication involved in Mullagulov experimentation. It should be noted that the destabilizing effect due to damping cannot be expected in short-duration tests (Mutyalarao et al., 2017).

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